MATH 501 – Analysis I Fall 2022-23 Instructor: Gökhan Göğüş

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We learn already in high school that integration plays a central role in mathematics and physics. One encounters usual Riemann integrals in the notions of area or volume, when solving a differential equation, in the fundamental theorem of calculus, in Stokes' theorem, or in classical and quantum mechanics. However, Riemann integral has certain disadvantages, in that some very basic functions are not Riemann integrable, that the pointwise limit of a sequence of Riemann integrable functions need not be Riemann integrable, and that the space of Riemann integrable functions is not complete with respect to the L^1 -norm. One purpose of this course is to introduce the Lebesgue integral, which does not suffer from these drawbacks and agrees with the Riemann integral whenever the latter is defined.

Lectures

Tuesday 14:40-16:30 FENS L056, Wednesday 12:40-13:30 FENS L029.

Textbook

R. L. Wheeden, A. Zygmund, Measure and integral. An introduction to real analysis, 2nd ed. (2015) Taylor & Francis Group, LLC. We will cover Chapters 1-8 and Chapter 10.

Recommended Reading

Rudin, Walter Real and complex analysis. Third edition. McGraw-Hill Book Co., New York, 1987. xiv+416 pp. ISBN: 0-07-054234-1

Rudin, Walter Principles of mathematical analysis. Third edition. International Series in Pure and Applied Mathematics. McGraw-Hill Book Co., New York-Auckland-Düsseldorf, 1976.

Course Outline

Lebesgue measure and integration on the line. Convergence theorems. General measure and integration. L^p spaces. Decomposition of measures. Radon Nikodym theorem. Product measures and Fubini's theorem.

Homework. During the semester you will be responsible for homework. All homework should be done individually. Your solutions of homework will be collected, and they will be graded. There is no make up for homework. The best 3 homework out of 4 will count as your homework grade; overall homework is 30% of your total grade. After each homework there will be an oral exam session related to the problems in the homework. You will get 80 percent from the solutions and 20 percent from the oral exam.

Tests/Grading

There will be one midterm, final exam, homework and oral exams.

| Best 3 Homework+Oral exam | 8%HW+2% Oral exam=10%each | 30% |
|---------------------------|--|-----|
| Midterm | 6 th December Tuesday, lecture time | 30% |
| Final | Date will be determined by SR | 40% |

Attendance: Students are strongly advised to attend all the lectures.

Makeup Policy

If you miss the midterm, you **must** contact me and explain your excuse as soon as possible. If it is a health problem you must bring your doctor's report, which is given or checked by SU Health Center, as well. In case you are unable to visit me, you, a friend or a relative should somehow (e-mail, phone, etc.) let me know about the situation.

Academic Honesty

The strength of the university depends on academic and personal integrity. In this course, you must be honest and truthful. Cheating hurts our community by undermining academic integrity, creating mistrust, and fostering unfair competition. The university will punish cheaters with failure on an assignment, failure in a course, permanent transcript notation, suspension, and/or expulsion. Violations can include cheating on exams, plagiarism, reuse of assignments without permission, improper use of the Internet and electronic devices unauthorized collaboration, alteration of graded assignments, forgery and falsification, lying, facilitating academic dishonesty, and unfair competition. Ignorance of these rules is not an excuse.

In this course, as in many math courses, working in groups to study particular problems and discuss theory is strongly encouraged. Your ability to talk mathematics is of particular importance to your general understanding of mathematics. You should collaborate with other students in this course on the general construction of homework assignment problems. However, you must write up the solutions to these homework problems individually and separately. If there is any question as to what this statement means, please see the professor or the recitation instructor.

For more information, see the guide on the SU website (http://mysu.sabanciuniv.edu/yonerge/Akademik_durustluk/E-Investigation.html)

Goals. By the end of this course, students should be able to:

(a) Use Caratheodery definition to construct Lebesgue-Stieltjes measure on the line and Lebesgue measure on any Euclidean space

(b) Define and use convergence theorems

(c) Define the Lebesgue Integral

(d) Use Radon Nikodym theorem, product measures and Fubini's theorem to evaluate integrals.

| Dates | Section Readings | HW/M/F |
|---------|--|--------------------|
| | (weekly) | |
| Week 1 | Ch. 1 & 2; topology of Rn, Riemann and | |
| | Riemann-Stietjes integrals | |
| Week 2 | Ch. 2 & 3; Further Riemann-Stietjes, | |
| | Lebesgue outer measures, Lebesgue | |
| | measure | |
| Week 3 | Ch. 3 & 4; Measurable functions, | |
| | semicontinuous functions | |
| Week 4 | Ch 4 & 5; Egorov's and Lusin's theorems, | |
| | Lebesgue integral | |
| Week 5 | Ch 5; Further Lebesgue integral | HW1 |
| Week 6 | Ch 6; Fubini's theorem | OE |
| Week 7 | Ch 6 & 7; Further Fubini, Lebesgue's | |
| | differentiation theorem | |
| Week 8 | Ch 7 & 8; Absolutely continuous | HW2 |
| | functions, Lp classes | |
| Week 9 | Ch 8, Further Lp classes, Minkowski's, | owski's, OE |
| | Hölder's, Schwartz inequalities | |
| Week 10 | Ch 10; Abstract measure and integration | HW3 |
| Week 11 | Ch 10; Abstract measure and integration | OE |
| Week 12 | Ch 10; Abstract measure and integration | |
| Week 13 | Ch 10; Abstract measure and integration | |
| Week 14 | | HW4&OE |